

Determining the Eccentricity of the Moon's Orbit without a Telescope, and Some Comments on "Proof" in Empirical Science

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ABSTRACT

Prior to the invention of the telescope many astronomers worked out theories of the motion of the Moon. The purpose of such theories was to be able to predict the *position* of the Moon in the sky. These geometrical models implied a certain range of distance of the Moon. Ptolemy's most quoted model, in fact, predicted that the Moon was nearly twice as far away at apogee than at perigee. Measurements of the angular size of the Moon were within the capabilities of pre-telescopic astronomers. These could have helped refine the models of the motion of the Moon, but hardly anyone seems to have made any measurements that have come down to us. Using a piece of cardboard with a small hole punched in it which slides up and down a yardstick, we show that it is possible to determine an approximate value of the eccentricity of the Moon's orbit. On the basis of 64 observations taken over 14 cycles of the Moon's phases we find $\epsilon \approx 0.041 \pm 0.004$. A typical measurement uncertainty of the Moon's angular size is ± 0.7 arcmin. Since the Moon's angular size ranges from 29.4 to 33.5 arcmin, carefully taken naked eye data are accurate enough to demonstrate periodic variations of the Moon's angular size.

Subject headings: lunar orbit, pre-telescopic astronomy

1. Introduction

The *small angle formula* is one of the simple geometrical relationships we discuss in elementary astronomy classes. An object of diameter d viewed at distance D will subtend an angle $\theta = d/D$ in radians if $D \gg d$. Two objects that satisfy this criterion are the Sun and Moon.

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The history and significance of estimating the angular diameter of the Moon is a curious one. We briefly review it here. Aristarchus of Samos (ca. 310–230 BC) wrote a treatise called *On the Sizes and Distances of the Sun and Moon*, which has survived.¹ In it Aristarchus states that the Sun and Moon must have about the same angular size, because during a total solar eclipse the Moon just barely covers the Sun. He also states that the angular size of the Moon is 2 degrees. In the *Sand Reckoner* Archimedes (ca. 287–212 BC) says that Aristarchus also used another value for the angular size of the Moon,² 1/720th part of a circle, or 0.5 deg.³ This is close to the true mean value of 0.5182 deg = 31′ 5″.⁴

Hipparchus (ca. 190–120 BC) also wrote a treatise on the sizes and distances of the Sun and Moon. This has not survived, but its contents are known from two sources.⁵ Hipparchus obtained a mean lunar distance of $67\frac{1}{3}$ Earth radii, with a range of 62 to $72\frac{2}{3}$. The range is ± 7.9 percent. In the *Almagest* (IV,9) Ptolemy (ca. 100–170 AD) quotes Hipparchus’s value for the Moon’s mean angular size of 1/650th of a circle, or 33′ 14″.⁶

Ptolemy’s values for the minimum and maximum angular size of the Moon were 31′ 20″ and 35′ 20″, respectively.⁷ But Ptolemy’s model for the *position* of the Moon in the sky implies that its minimum distance is a mere 33.55 Earth radii, while its maximum distance is 64.17 Earth radii.⁸ Given the importance of the *Almagest* from ancient times until the

¹Thomas Heath, *Aristarchus of Samos: the Ancient Copernicus* (Clarendon Press, Oxford, 1913), pp. 351-414.

²Heath, note 1, pp. 353, 383.

³Albert Van Helden, *Measuring the Universe: Cosmic Dimensions from Aristarchus to Halley* (Chicago and London: University of Chicago Press, 1985), pp. 8-11.

⁴Arthur N. Cox, ed. *Allen’s Astrophysical Quantities* (Springer-Verlag, New York, 2000), pp. 308-309. Two times the “mean Moon radius” (1738.2 km) divided by the mean distance (384,401 km), times $180/\pi = 0.5182$ deg = 31.09 arcmin. Given the extreme range of the Moon’s distance (356,400 to 406,700 km), its geocentric angular size ranges from 29.39 to 33.53 arcmin. The mean distance of the Moon divided by the equatorial radius of the Earth, 6378 km, gives a mean geocentric distance of 60.27 Earth radii. Note that the maximum distance is 5.8 percent greater than the mean distance, and the minimum distance is 7.3 percent less than the mean distance.

⁵G. J. Toomer, “Hipparchus on the distances of the Sun and Moon,” *Archive for the History of Exact Sciences*, **14**, 126–142 (1975).

⁶G. J. Toomer, *Ptolemy’s Almagest* (Berlin, Heidelberg, Tokyo: Springer-Verlag, 1984), p. 205.

⁷Toomer, note 6, pp. 254, 284.

⁸Toomer, note 6, pp. 251, 259. In section 5.13 of the *Almagest* Ptolemy gives the mean distance of the Moon at syzygy (i.e. new/full Moon) of 59 Earth radii. The mean distance at quadrature (i.e. first or third quarter) is $38\frac{43}{60}$ Earth radii, and the radius of the epicycle is $5\frac{10}{60}$ Earth radii. It follows that the greatest

Renaissance, the implication that the Moon’s distance (and, hence, angular size) ranges by nearly a factor of two was known to all those who worked on models of the Moon’s motion, though any serious observer of the Moon would have known that its range of angular size is considerably smaller.

Simplicius (6th century AD), in his commentary on Aristotle’s *De Caelo*, wrote: “...if we observe the moon by means of an instrument ... it is at one time a disk of eleven finger-breadths, and again at another time a disk of twelve finger-breadths.” Here a “finger-breadth” (digit, or *daktylos*) is *not* the angular size of a finger at some distance. It is a Babylonian unit equal to one-twelfth of a *degree*, but in this context it might be one-twelfth of the maximum angular size of the Moon. Taken at face value, the statement by Simplicius implies the existence of observations that give just about the right range of the Moon’s angular size. But the data and the identity of the observer are not given.⁹

Finally, in the fourteenth century two astronomers showed an interest in actually making some measurements. Levi ben Gerson (1288-1344), also known as Gersonides, was a considerably versatile and accomplished scholar.¹⁰ He invented the staff of Jacob, which consists of a calibrated ruler that slides perpendicularly along another calibrated staff. With it one could determine the angular separation of two stars in the sky or determine the height of building. Using a calibrated staff and a pinhole camera, he determined that the Sun ranges in angular size from 27′ 50″ to 30′ 0″.¹¹ He commented on Ptolemy’s factor-of-two range of the Moon’s implied angular size. Using the staff and pinhole camera, Levi found a lunar diameter at quadrature only slightly larger than at opposition (i.e. full Moon).¹² He did *not* carry out observations at all lunar phases.

Another pre-telescopic astronomer to comment specifically on the range of lunar angular size as a consequence of a model to explain its varying position was Ibn al-Shatir (1304–1375/6 AD).¹³ The implied range was 29′ 2″ to 37′ 58″.

distance occurs at syzygy and is $59 + 5\frac{10}{60} = 64.17$ Earth radii. The minimum distance occurs at quadrature and is $38\frac{43}{60}$ minus $5\frac{10}{60} = 33.55$ Earth radii. See also note 17 below.

⁹Thomas L. Heath, *Greek Astronomy* (Dover, New York, 1991), pp. xvii, 69. See also the anonymous article, “Babylonian measures and the *daktylos*,” *Observatory*, **42**, 46–51 (1919).

¹⁰Bernard R. Goldstein, *The Astronomy of Levi ben Gerson (1288–1344)* (New York: Berlin, Heidelberg, Tokyo: Springer-Verlag, 1985).

¹¹Goldstein, note 10, p. 113.

¹²Goldstein, note 10, pp. 105, 186.

¹³V. Roberts, “The solar and lunar theory of Ibn ash-Shatir: a pre-Copernican model,” *Isis*, **48**, 428–432

Regiomontanus (1436–1476) doubted the very large range of lunar angular size implied by Ptolemy’s model, basing his criticism on al-Battani’s apparent diameters at syzygy.¹⁴ In *De Revolutionibus* Copernicus (1473–1543) gives a range of 30′ to 35′ 38″ for the Moon’s angular size.¹⁵

Tycho Brahe (1546–1601), the greatest of all pre-telescopic astronomers, devoted considerable effort to the orbit of the Moon, but he was concerned only with the Moon’s varying position. Still, his model implied a range of lunar distance of 5.8 percent at syzygy,¹⁶ or ± 2.9 percent.

Of course, we now can model the orbit of the Moon quite accurately. But we should not simply say that the eccentricity of the Moon is $\epsilon = 0.05490$,⁴ implying a range of distance of ± 5.5 percent. Contrary to what introductory astronomy textbooks say, the Earth does not orbit the Sun on a simple elliptical orbit, and the Moon does not orbit the Earth on a simple elliptical orbit either. The Earth-Moon barycenter orbits the Sun on a nearly Keplerian ellipse. The Earth and Moon do something else. Let $\delta = 0.011$ be the “amplitude of Ptolemy’s evection.” To first order the maximum deviation from uniform angular motion of the Moon varies from $\pm 2(\epsilon - \delta)$ when the Moon is new or full and $\pm 2(\epsilon + \delta)$ for first and third quarters. The distance of the Moon (to first order) varies by $(\epsilon + \delta) = \pm 6.6$ percent.¹⁷ Three modern ephemerides use 669, 921, and 915 terms, respectively, to calculate the distance to the Moon!¹⁸

One might think that Hipparchus or some other pre-telescopic astronomer had actually

(1957).

¹⁴Johannes Regiomontanus, *Epytoma in almagesti Ptolemei* (Venice, 1496), §5.22. There is no published translation of this work. al-Battani lived from about 858 to 929 AD.

¹⁵Edward Rosen, *Nicholas Copernicus: On the Revolutions* (Baltimore and London: Johns Hopkins Univ. Press, 1992), p. 209.

¹⁶N. Swerdlow, “The lunar theories of Tycho Brahe and Christian Longomontanus in the *Progymnasmata* and *Astronomia Danica*, *Annals of Science*, **66**, 5–58 (2009), on p. 43.

¹⁷M. C. Gutzwiller, “Moon-Earth-Sun: the oldest three-body problem,” *Reviews of Modern Physics*, **70**, 589–639 (1998), pp. 601–602. Compared to uniform motion against the background of stars, the new/full Moon can be 5 deg ahead or behind. At first/third quarter the Moon can be 7.5 deg ahead or behind the mean motion. To quote Gutzwiller: “This new feature is known as the *evection*. Ptolemy found a mechanical analog for this peculiar complication, called the crank model. It describes the angular coupling between the Sun and Moon correctly, but it has the absurd consequence of causing the distance of the Moon from the Earth to vary by almost a factor of 2.”

¹⁸Gutzwiller, note 17, p. 628.

measured the variation of the apparent angular size of the Moon over a number of lunations. Even if this is true, no data set or analysis of one has come down to us. With an instrument as simple as a quarter-inch diameter sighting hole viewed at some distance down a yardstick, is it possible to demonstrate that the Moon’s angular size varies from 29 to 33 arcmin in a quasi-sinusoidal way?

2. Observations

Hipparchus measured the angular size of the Moon using a *dioptra*. Such a device uses a round object of small angular size to *occlude* the Moon. As did Levi ben Gerson,¹⁸ we have found that it is better to use a sighting hole.

We fashioned a cross piece of cardboard that can slide up and down a yardstick. The yardstick is calibrated in centimeters on one edge. There are tick marks every 5 mm. The purpose of the cross piece is simply to hold a thin piece of cardboard that has a small hole punched through it. We estimate that the diameter of the sighting hole is 6.2 mm.

In Table 1 we present a series of observations carried out during 2009. All the data were taken at my home in College Station, Texas, which is at an elevation of about 100 m above sea level. All observations were made with my left eye, which is my better eye. In Table 1 the age of the Moon is the number of days since the previous new Moon.¹⁹ The “true angular size” is the geocentric angular diameter interpolated from values given in the *Astronomical Almanac*.²⁰ Observations from 11:15 to 13:15 UT and from 22:37 to 23:43 UT were made during twilight or daytime. This constituted a majority of the observations. Starting on 04 August 2009 our data values in column 5 of Table 1 are typically the average of two measurements, the first obtained while moving the cross piece *out* from a position much too close to the eye, the second obtained by moving the cross piece *in* from the far end of the yardstick. A pair of such observations made on the same occasion typically exhibits a difference of 8 mm. So a typical internal error of one of the averages in column 5 is ± 4 mm.

Let D be the distance down the yardstick that the 6.2 mm sighting hole is viewed. Using the small angle approximation one obtains the observed angular size of the Moon as follows:

$$\theta_{\text{obs}}(\text{arcmin}) = \left(\frac{6.2}{D} \right) \times \left(\frac{180}{\pi} \right) \times 60 . \quad (1)$$

¹⁸Goldstein, note 10, p. 156.

¹⁹aa.usno.navy.mil/data/docs/MoonPhase.php#y2009

²⁰*The Astronomical Almanac for the Year 2009* (Washington, D. C., Nautical Almanac Office, 2009).

Taken at face value, our mean angular size is 25.9 arcmin. Thus, our preliminary data exhibit a systematic error of 5.2 arcmin. However, consider that the pupil has a non-zero size. In fact, the pupil’s size is comparable to the size of the sighting hole.

We therefore devised a simple calibration. A disk of diameter 90.44 mm would subtend an angle of 31.09 arcmin if situated at a distance of 10 m. We cut out a disk measured to be 91 mm in diameter. Situated at 10 m, the angular size was 31.28 arcmin. If it were not for the finite size of the pupil, a 6.2 mm hole viewed at 681.3 mm would subtend an equal angle. If we place the sighting hole at this distance, we can see beyond the left side of the disk from the right side of the pupil, and we can see beyond the right side of the disk from the left side of the pupil. So we need to place the sighting hole at a *greater* distance from the eye to make the best apparent fit of the disk as seen through the hole. I found that I had to place the sighting hole 821 mm from my left eye to produce the best match to the angular size of the 91 mm disk. (This calibration was done at the office on 5 August 2009 in an illuminated hallway.) We thus obtain a correction factor of $821/681.3 = 1.205$, which should be used to scale our preliminary angular sizes. In other words, from April through November 2009 when these lunar observations were made, for my eye

$$\theta_{\text{corr}} = \theta_{\text{obs}} \times 1.205 . \quad (2)$$

Further justification of the correction factor is shown in Fig. 2, in which we plot the ratios of true (geocentric) angular sizes to the corresponding observed values. One outlier is eliminated from analysis. A simple regression line gives a non-zero slope at the $2.3\text{-}\sigma$ level of significance. Thus, there is marginal (but not statistically significant) evidence that I was measuring the Moon larger over time. The average value of the ratios is 1.199 ± 0.005 , which is very close to the correction factor derived from a measurement of the 91 mm disk at 10 m. Ideally, one would do this calibration after each measurement of the Moon under lighting conditions as similar as possible. Maybe there is a sufficiently good set of three correction factors for each observer, one for nighttime, one for twilight, and one for daytime observations. But this is beyond the scope of the present paper.

We note that the correction factor eliminates any systematic error in the adopted size of the sighting hole. If the true diameter of the sighting hole were really 6.4 mm, then the correction factor would be correspondingly smaller. For the calibration given above and observations with my left eye, with minimal error for the Moon’s varying angular size I could use the following:

$$\theta_{\text{corr}} = \left(\frac{821}{D} \right) \times 31.28 . \quad (3)$$

Thanks to the correction factor, our mean value for the angular size is 31.18 arcmin, very close to the true mean value of 31.09.

3. Analysis and Discussion

Even before the ancient Greeks, the Babylonians knew the anomalistic period of the Moon. This is the time between occurrences of maximum daily motion against the background of stars (i.e. the time from perigee to perigee), 27.55455 days.⁴ We have taken data over 7 lunations = 7.5 anomalistic months. Ideally, one would have a data set that extends over 14 lunations = 15 anomalistic months. In that case every phase would occur at some time over the range of observations at perigee and at apogee.

Since all observations presented in Table 1 were made by the same observer with the same eye and the same sighting hole, we could derive a value of the eccentricity of the Moon’s orbit from an analysis of the distances-down-the-yardstick given in column 5 of the table, or we could use the uncorrected (observed) angular sizes of the Moon. The eccentricity of the orbit is just the amplitude of the sinusoid that best fits the data, divided by the mean value of the data. But we choose to use the corrected angular sizes given in column 6 of Table 1 because the use of the correction factor eliminates a significant source of systematic error.

We note that while a Keplerian elliptical orbit is necessarily eccentric, without more accurate data than we present here we cannot prove that the Moon’s orbit is elliptical, circular, ovoid, or some other shape. We have only demonstrated that the Earth does not reside at the center of the Moon’s orbit, as was known by Ptolemy, Copernicus, and others.

Say we knew nothing of the value of the anomalistic month. What does our data set tell us is the time between perigees? We have eliminated one data point from the analysis, the first datum from October 23rd, which is a $4.5\text{-}\sigma$ outlier. Using a program for the analysis of variable star light curves,²¹ we obtain a best fit period of 27.24 ± 0.29 days. The uncertainty of the period is to be considered a lower limit.²² The amplitude of the variation is 1.23 ± 0.17 arcmin. The root-mean-square scatter of the data with respect to that sinusoid is ± 0.74 arcmin, which is the internal random error of the measurements. The implied eccentricity of the Moon’s orbit is 0.039 ± 0.006 . In the top graph of Fig. 3 we show our data folded with the best period derivable from only our data set.

The bottom graph of Fig. 3 shows the geocentric angular size of the Moon (from the *Astronomical Almanac*) at the dates and times we took data, folded with the anomalistic period of the Moon. Note that the actual “true values” do not exhibit a simple sinusoid.

²¹M. Breger, “PERDET: multiple PERiod DETermination User Manual,” Communications in Astroseismology, No. 6 (1989).

²²M. H. Montgomery, D. O’Donoghue, “A derivation of the errors for least squares fitting to time series data,” Delta Scuti Star Newsletter, No. 13, 28-32 (1999).

Comparison of our corrected angular sizes to the geocentric values from the *Astronomical Almanac* indicates that an individual data point of ours is accurate to ± 0.80 arcmin. This is only marginally larger than the internal random error, implying that the corrected angular sizes contain no serious systematic offset. One way to slightly improve the analysis would be to correct the “true” angular sizes based on the geocentric distance of the Moon to the topocentric values.

The data obtained when the age of the Moon is between 7.7 and 22.4 days (i.e. first quarter to third quarter) are a bit more accurate than observations of the crescent Moon. We believe that twilight and daytime observations are more accurate than nighttime observations, because the Moon’s glare makes the observations more difficult.

It is remarkable that none of the great pre-telescopic astronomers carried out simple observations like those presented here. Of course, before the 19th century there was no Fourier analysis or least-squares theory to derive some of the numbers presented here, but the Babylonians *did* know the length of the anomalistic month to a high degree of accuracy, so measurements of the angular diameter of the Moon could have been folded with that period using nothing more than simple arithmetic. It could be asserted that because no known pre-telescopic data set has come down to us, the analysis presented here is completely unhistorical, and therefore of no interest to historians. Perhaps this paper will motivate scholars who can read Arabic or Hebrew to identify and scrutinize some previously unstudied manuscripts that contain data like those presented here.

Granted, the geometrical models of the Moon’s motion were concerned with its ecliptic latitude and longitude, not with its distance. But observations of the Moon’s angular size, which is to say measures related to its physical distance, could have nudged astronomers down the path of more physically realistic models of the solar system.

I thank Dudley Herschbach for suggesting that I write this article. I am particularly grateful to Noel Swerdlow for numerous fruitful discussions. Alexei Belyanin, Gerald Handler, and an anonymous referee provided useful references. I also thank James Evans and Stephen O’Meara for useful comments.

A. Additional Data and Some Commentary

In this appendix we provide some additional data obtained since mid-November, 2009. We also give some philosophical comments not included in the paper published in the *American Journal of Physics* in August of 2010.

In Table 2 we give an additional 29 measurements of the Moon. The average ratio of the true (geocentric) angular size to the observed angular size is 1.158 ± 0.006 for these measures. Eliminating the one outlier from 23 October 2009, the average ratio of “true” to “observed” angular size is 1.18 for 64 good measurements.

Our data set now covers 14 cycles of the Moon’s phases and 15 cycles of its anomalistic period. Thus, all waxing and waning phases of the Moon occurred at apogee and perigee over this time period.

In Table 3 we give seven measures of the correction factor C . The first three were made during the daytime in lighted hallways at Texas A&M University. The first and third measures were made at the same location, but the measure of 27 January 2010 was made elsewhere. The final four measures were made during twilight at the author’s home, in an attempt to calibrate the cross staff under conditions as similar as possible to many of the observations of the Moon made during twilight.

It seems that my correction factor was not a constant over the course of the 14 months of these observations. Exactly why I cannot say, but at age 56 my eyes are not what they once were. Perhaps my left eye was changing measurably over the past year, given this evidence. It has been suggested that maybe the way to get around the potential of a variable correction factor is to use a 1 mm sight (or smaller) at the eye end of the yardstick.

What, then, is the best way to demonstrate the sinusoidal variation of the angular size of the Moon? In the world of variable star observing one could have a secular variation of brightness of a star and cyclical changes on top of that. For our whole data set we fit a fourth order polynomial to the uncorrected angular sizes, then subtracted that curve. This would take care of any slow trends in the actual correction factor over time. The mean value of θ_{obs} is 26.38 arcmin. Since the true mean angular size of the Moon is 31.09 arcmin, that is also confirmation that our true mean correction factor must be about 1.18.

In Fig. 4 we have eliminated one outlier (the first observation of 23 October 2009) and have folded the residuals of θ_{obs} with a revised period of 27.5854 ± 0.0955 d. Note that this is only 0.03 d longer than the true known anomalistic period of the Moon. We find an epoch of apogee (i.e. minimum angular size) of Julian Date 2,454,965.8794, or 14 May 2009 at 9 hours UT. This is precisely one date of minimum angular size given in the 2009 volume of

the *Astronomical Almanac*.

Based on 64 measurements we find the eccentricity of the Moon’s orbit to be $\epsilon = (1.0853 \pm 0.109)/26.38 = 0.041 \pm 0.004$, just about what we obtained from the first 35 good data points. On the basis of our naked eye observations, the corrected amplitude of the sinusoid that describes the variation of the Moon’s angular size is $A = 1.18 \times 1.0853 \approx 1.28$ arcmin. The average random error of the 64 measurements is ± 0.72 arcmin. The first measurement from 23 October 2009 can be shown to be a $3.3\text{-}\sigma$ outlier even without any information of the true angular size of the Moon.

Using the methods described in this paper, students in my Astronomy 101 class made 34 measurements of the Moon on 22 February 2010 UT. The median value was 31.35 arcmin, exactly equal to the true angular size on that night! However, the standard deviation of their measures was ± 4.1 arcmin. I suspect that a novice observer could not measure the variation of the Moon’s angular size very well. The internal random error has to be less than 1.0 arcmin to succeed. Even I practiced for 4 months making observations with my worse eye until I realized I had to start over from scratch using my better eye. And I found that twilight observations are usually the best. In the upper graph of Fig. 4 the triangles make a tighter sinusoid than the dots.

One of the motivations of this paper was to provide an example to our students of the scientific method. As outlined in many an astronomy text,²³ we observe something in the laboratory or in the universe and we try to explain what we see. A good hypothesis allows us to make predictions. We can then make some measurements. If the data agree with the predictions, that helps confirm the hypothesis. If the data do not agree with the predictions, then the data might be wrong somehow, maybe the hypothesis needs modification, or maybe we need a completely new hypothesis.

My experience is that when students hear about Kepler’s Laws of planetary motion they only try to memorize them for upcoming quizzes and tests. These laws are quite abstract to the average non-astronomer. If we can get the students to confront some data, that should allow them an understanding at a deeper level.

So here is one hypothesis: though the Moon is not a planet, it orbits the Earth (or, more exactly, the Earth-Moon barycenter) on an elliptical orbit having an eccentricity of a few percent. If we measure the angular size of the Moon, and the data show no statistically significant changes of the Moon’s size, then the hypothesis might be wrong, or the data

²³See, for example: Jeffrey Bennett, Megan Donahue, Nicholas Schneider, and Mark Voit, *The Essential Cosmic Perspective*, 5th ed. (San Francisco, Addison-Wesley), 2009, p. 70 ff.

might not be accurate enough to demonstrate the effect. If the data show regular ± 4 percent variations of the Moon’s angular size, like our data in Tables 1 and 2 and shown in Figs. 3 and 4, then under the assumption that the Moon’s *linear* size is constant (i.e. that the Moon does not pulsate), we have evidence that the Moon’s distance from the Earth varies ± 4 percent. I assert that this is *confirming evidence* that Kepler’s First Law holds for the Moon. As we state in the third paragraph of §3 above, our data are not accurate enough to prove that the orbit is elliptical, circular, ovoid, or some other shape. But we have *proven* that the Earth is not at the center of the Moon’s orbit.

The *American Journal of Physics* uses two referees for each paper. One of our original two referees said that the first submitted version of this paper was acceptable for publication. The second referee was an historian and had some serious reservations. The second referee absolutely would not allow me to say that we have confirming evidence for Kepler’s First Law of orbital motion. That referee and then a third referee would not let me state that the data *prove* that the Earth is offset from the center of the Moon’s orbit. I was required to use either of the verbs “show” or “demonstrate”.

Admittedly, the words “prove” and “proof” are seldom used in scientific papers.²⁴ And of course, in an empirical science such as astronomy, we would not use it in the same way as in a mathematical or geometrical proof, which must be absolutely true given certain starting assumptions. Still, in astronomy, we can say we have proven something. Much more common, and less strong, is to say that we have “evidence for” something, or that the data are “consistent with” some hypothesis. A still weaker claim is to say that the data are “not inconsistent with” some hypothesis. For example, if two measures should be the same but differ by three standard deviations, then we might say that the two results are not inconsistent with each other. We feel more confident with the agreement if the two numbers are within one standard deviation. Then we say that the two numbers are statistically equal.

Under the *assumption* that a single sine wave fits the variations of our corrected angular sizes of the Moon, we did not quite find $\epsilon = 0.055$. Owing to the joint gravitational forces

²⁴See: Douglas Clowe, Maruša Bradač, Anthony H. Gonzalez, Maxim Markevitch, Scott W. Randall, Christine Jones, and Dennis Zaritsky, “A direct empirical proof of the existence of Dark Matter,” *Astrophysical Journal*, **648**, L109–L113 (2006). One time I really wanted to use the word “proof” but opted for “strong evidence”. See: Kevin Krisciunas, R. F. Griffin, E. F. Guinan, K. D. Luedeke, and G. P. McCook, “9 Aurigae: strong evidence for non-radial pulsations,” *Monthly Notices of the Royal Astronomical Society*, **273**, 662–674 (1995). We *did* prove the existence of a new class of non-radially pulsating stars. See also: Anthony B. Kaye, Gerald Handler, Kevin Krisciunas, Ennio Poretti, and Filippo M. Zerbi, “ γ Doradus stars: defining a new class of pulsating variables,” *Publications of the Astronomical Society of the Pacific*, **111**, 840–844 (1999).

of the Earth and Sun, the actual orbit of the Moon is quite complicated. Its minimum distance is 7.3 percent closer than the mean distance, and its maximum distance is 5.8 percent greater than the mean distance. But we did prove/show/demonstrate that pre-telescopic astronomers could have generated a data set like ours presented here. It would be interesting to investigate when “consistency checks” became a part of the modern method of doing science. Had the pre-telescopic astronomers used angular size measurements to make consistency checks of their models of the Moon’s motion, they would have easily shown that Ptolemy’s most quoted model could not be physically correct. One main point of the Copernican revolution, as I understand it, is that by the end of the 16th century we were ready to go beyond geometrical models that merely accounted for the varying positions of the Moon, Sun, and planets against the background of stars. We were ready to determine the true, physical arrangement of objects in the solar system.

Table 1. Lunar Data

2009 UT Date	UT	JD ^a	Age ^b	D ^c	θ_{corr}	θ_{true} ^d
21 Apr	11:23	4942.9743	25.80	874.0	29.39	30.69
06 May	03:55	4957.6632	11.02	840.0	30.58	31.03
29 May	01:33	4980.5646	4.56	810.0	31.71	32.52
31 May	01:33	4982.5646	6.56	794.0	32.35	31.82
14 Jul	11:15	5026.9688	21.65	828.0	31.02	30.65
16 Jul	12:02	5029.0014	23.69	812.0	31.63	31.65
29 Jul	01:33	5041.5646	6.96	816.0	31.47	30.62
04 Aug	02:08	5047.5889	12.98	859.0	29.90	29.42
07 Aug	12:01	5051.0007	16.39	846.5	30.34	29.72
10 Aug	11:58	5053.9986	19.39	846.5	30.34	30.40
13 Aug	12:04	5057.0028	22.40	796.0	32.27	31.48
14 Aug	11:57	5057.9979	23.39	817.0	31.44	31.90
15 Aug	12:08	5059.0056	24.40	816.0	31.47	32.31
17 Aug	10:00	5060.9167	26.31	805.0	31.90	32.96
27 Aug	02:48	5070.6167	6.70	810.0	31.71	30.26
30 Aug	03:00	5073.6250	9.71	870.5	29.50	29.70
02 Sep	01:12	5076.5500	12.63	899.0	28.57	29.56
07 Sep	02:48	5081.6167	17.70	811.0	31.67	30.83
09 Sep	11:45	5083.9896	20.07	795.0	32.31	31.23
16 Sep	11:46	5090.9903	27.07	804.0	31.94	32.80
27 Sep	23:30	5102.4792	9.20	848.0	30.29	29.54
29 Sep	23:42	5104.4875	11.21	865.0	29.69	29.68
03 Oct	03:51	5107.6604	14.38	859.5	29.88	30.41
08 Oct	03:40	5112.6528	19.37	841.0	30.54	31.73
08 Oct	13:00	5113.0417	19.76	802.0	32.02	31.81
15 Oct	11:54	5119.9958	26.72	761.5	33.73	32.23
23 Oct	01:27	5127.5604	4.83	765.0	33.57	29.95
23 Oct	23:29	5128.4785	5.75	840.0	30.58	29.74
24 Oct	23:42	5129.4875	6.76	843.0	30.47	29.60
25 Oct	22:37	5130.4424	7.71	868.0	29.59	29.56
27 Oct	23:43	5132.4882	9.76	819.0	31.36	29.75

Table 1—Continued

2009 UT Date	UT	JD ^a	Age ^b	D ^c	θ_{corr}	θ_{true} ^d
31 Oct	23:28	5136.4778	13.75	817.0	31.44	30.98
03 Nov	12:51	5139.0354	16.30	792.0	32.43	31.85
05 Nov	12:42	5141.0292	18.30	773.0	33.23	32.26
06 Nov	12:55	5142.0382	19.31	789.5	32.53	32.35
10 Nov	13:15	5146.0521	23.32	800.0	32.10	32.11

^aJulian Date *minus* 2,450,000.

^bTime in days since previous new Moon.

^cDistance from the eye (in mm) of the 6.2 mm diameter sighting hole. Typical internal error is ± 4 mm.

^dGeocentric angular diameter of the Moon (in arcmin), interpolated from data given in the *Astronomical Almanac*.

Table 2. Additional Data^a

2009/2010 UT Date	UT	JD	Age	D	θ_{obs}	θ_{true}
26 Nov	23:40	5162.4861	10.18	791.0	26.95	30.31
04 Dec	13:05	5170.0451	17.74	777.0	27.43	32.85
04 Dec	14:16	5171.0944	18.79	760.0	28.04	32.79
09 Dec	14:11	5175.0910	22.79	804.0	26.51	31.79
20 Dec	23:08	5186.4639	4.46	840.0	25.37	29.45
25 Dec	23:37	5191.4840	9.48	777.0	27.43	30.65
27 Dec	23:04	5193.4611	11.46	786.0	27.12	31.63
28 Dec	23:20	5194.4722	12.47	793.0	26.88	32.16
30 Dec	23:34	5196.4819	14.48	748.5	28.48	33.00
21 Jan	23:38	5218.4847	6.69	817.5	26.07	30.16
26 Jan	23:18	5223.4708	11.67	781.5	27.27	32.53
06 Feb	13:57	5234.0812	22.28	756.0	28.19	30.67
22 Feb	00:03	5249.5021	7.88	763.5	27.92	31.35
28 Feb	00:24	5255.5167	13.90	745.0	28.61	33.38
28 Feb	04:28	5255.6861	14.07	784.0	27.19	33.36
25 Apr	00:49	5311.5340	10.51	774.5	27.52	32.53
02 May	12:16	5319.0111	17.99	824.0	25.87	30.42
23 May	01:07	5339.5465	9.00	760.0	28.04	32.15
28 May	02:44	5344.6139	14.07	764.0	27.90	30.99
30 May	04:38	5346.6931	16.15	788.0	27.05	30.34
30 May	11:10	5346.9653	16.42	798.0	26.71	30.26
30 May	12:08	5347.0056	16.46	819.0	26.02	30.25
31 May	11:42	5347.9875	17.44	808.0	26.38	29.98
04 Jun	11:56	5351.9972	21.45	838.0	25.43	29.59
22 Jun	01:16	5369.5528	9.58	789.0	27.01	31.42
23 Jun	01:39	5370.5688	10.60	790.0	26.98	31.14
24 Jun	01:41	5371.5701	11.60	795.0	26.81	30.87
28 Jun	11:32	5375.9806	16.01	835.0	25.53	29.80
04 Jul	13:12	5382.0500	22.51	830.0	25.68	29.92

^aThe meanings of the column labels are the same as in Table 1 except column 6 contains the *observed* (i.e., uncorrected) angular sizes, not the corrected ones.

Table 3. Measures of the Correction Factor

UT Date	UT	JD ^a	C
05 Aug 2009	...	5049.25	1.205
27 Jan 2010	...	5224.33	1.151
29 Jan 2010	...	5226.31	1.118
06 Feb 2010	14:07	5234.09	1.118
05 May 2010	01:05	5321.55	1.083
28 Jun 2010	11:39	5375.98	1.072
06 Jul 2010	01:34	5383.56	1.086

^aJulian Date *minus* 2,450,000.

Fig. 1.— A simple Moon measuring device, made from the bottom of a box of pancake mix.

Fig. 2.— Ratio of true (geocentric) angular sizes of the Moon to the corresponding measured (i.e. uncorrected) values.

Fig. 3.— Corrected values of Moon’s angular size and true geocentric values.

Fig. 4.— Phased measures of the variation of the Moon’s angular size, both unbinned and binned. One outlier has been eliminated from the analysis.

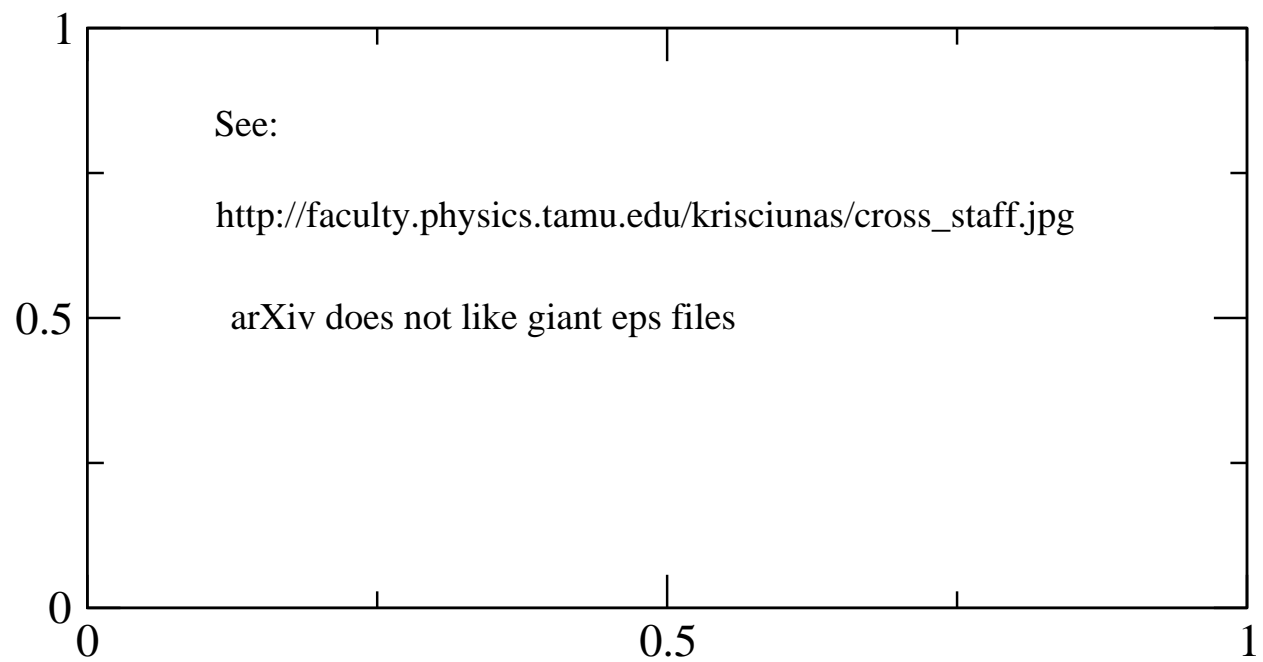


Fig. 1. Link to graphic showing a simple Moon measuring device.

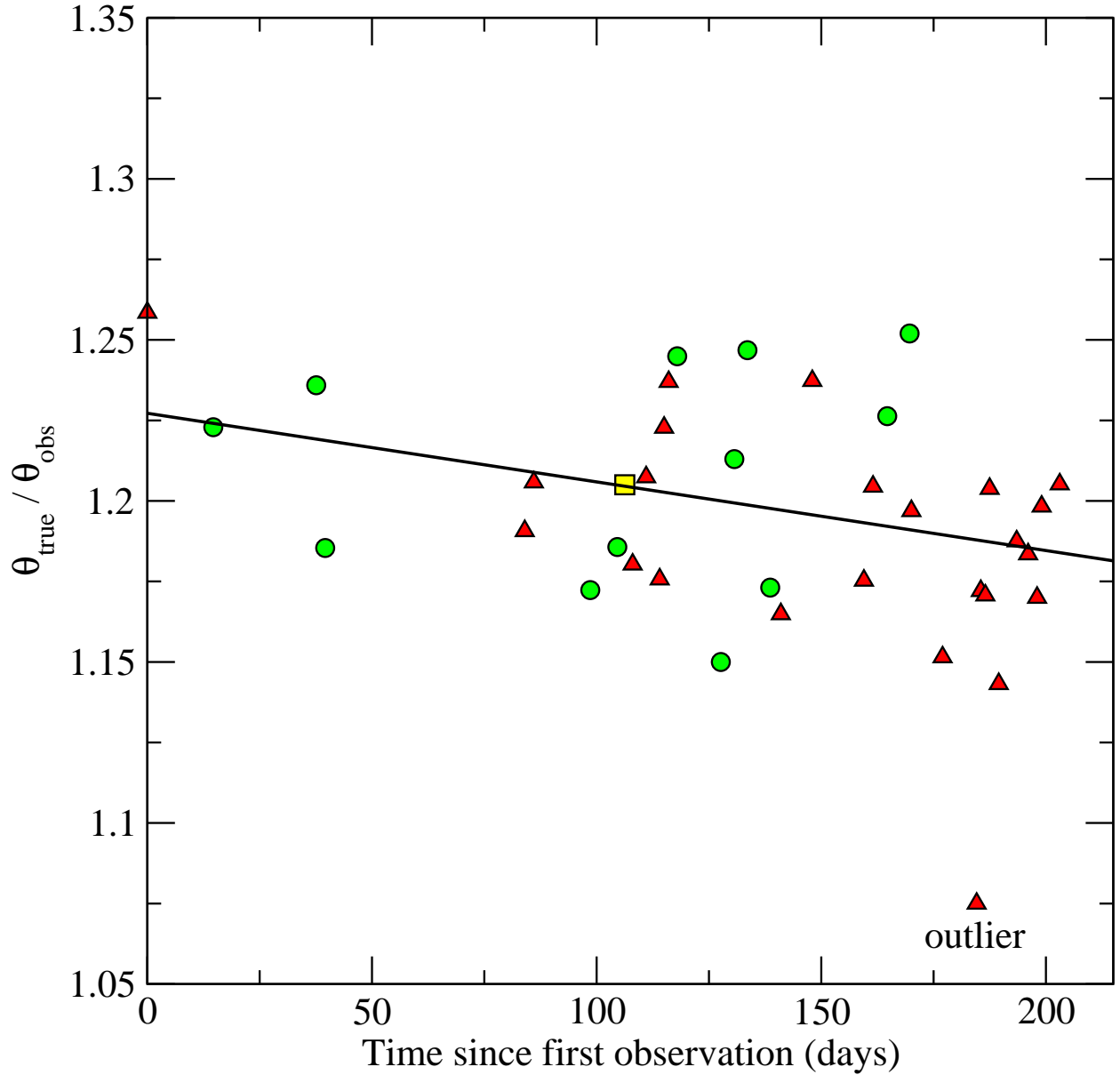


Fig. 2. Ratios of the true (geocentric) angular sizes of the Moon to the corresponding measured (i.e. uncorrected) values. Data represented by triangles use observations made during twilight or daytime. Data represented by dots use observations made at nighttime when the observer's pupil was presumably smaller. The square represents a calibration done with a 91 mm disk viewed at 10 m, which matches the true mean angular size of the Moon. The slope of the regression line is non-zero at the $2.3\text{-}\sigma$ level, which is to say it is not statistically significantly different than zero.

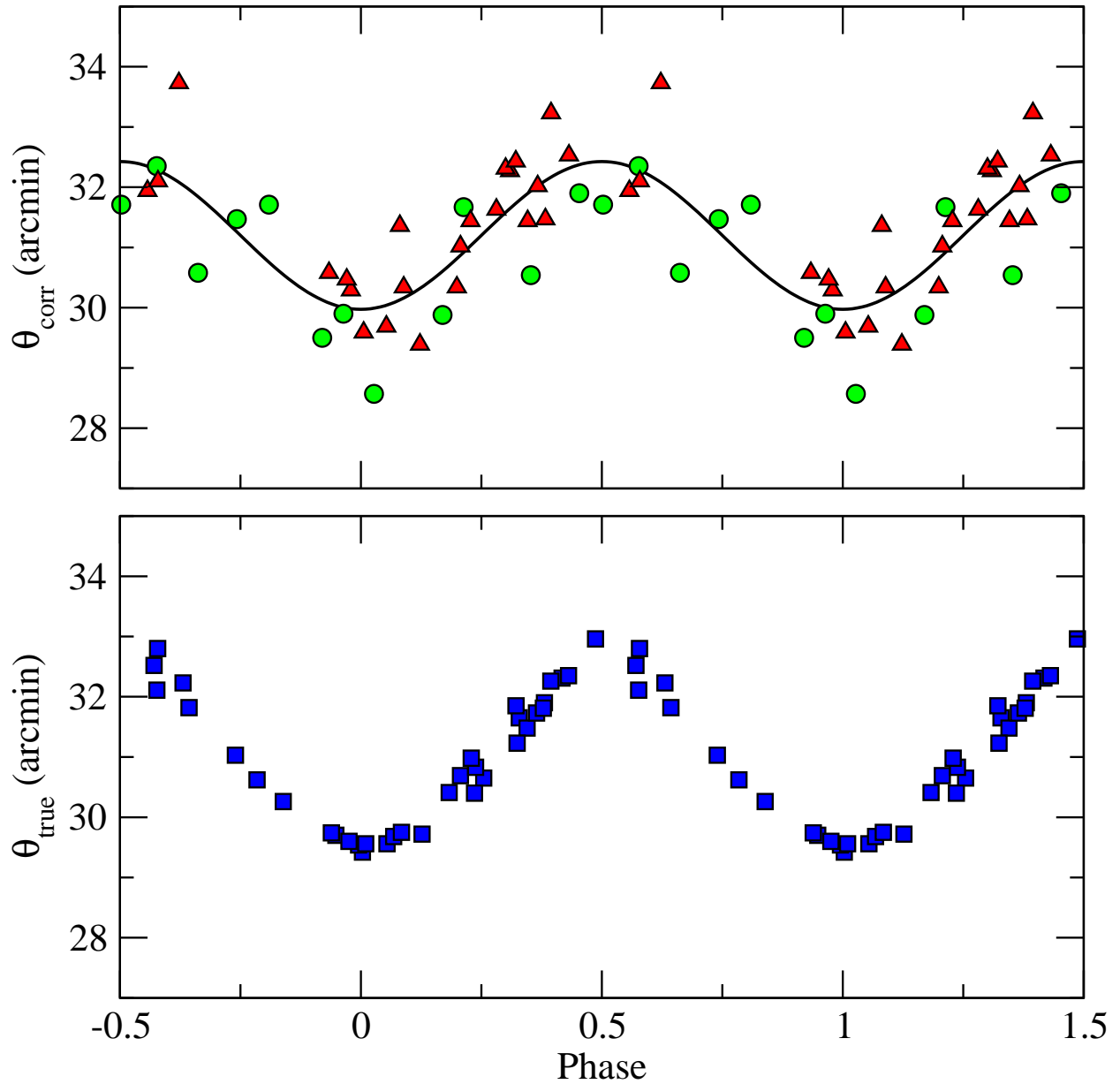


Fig. 3. Angular diameter of the Moon vs. phase in its cycle from apogee to apogee. *Top:* Corrected values from 21 April through 10 November 2009. Data represented by triangles were made during twilight or daytime. Data represented by dots were made at nighttime. The solid line is a single sine wave whose amplitude and mean value were given by program PERDET. *Bottom:* True geocentric values of the Moon's angular size, as interpolated from the *Astronomical Almanac*.

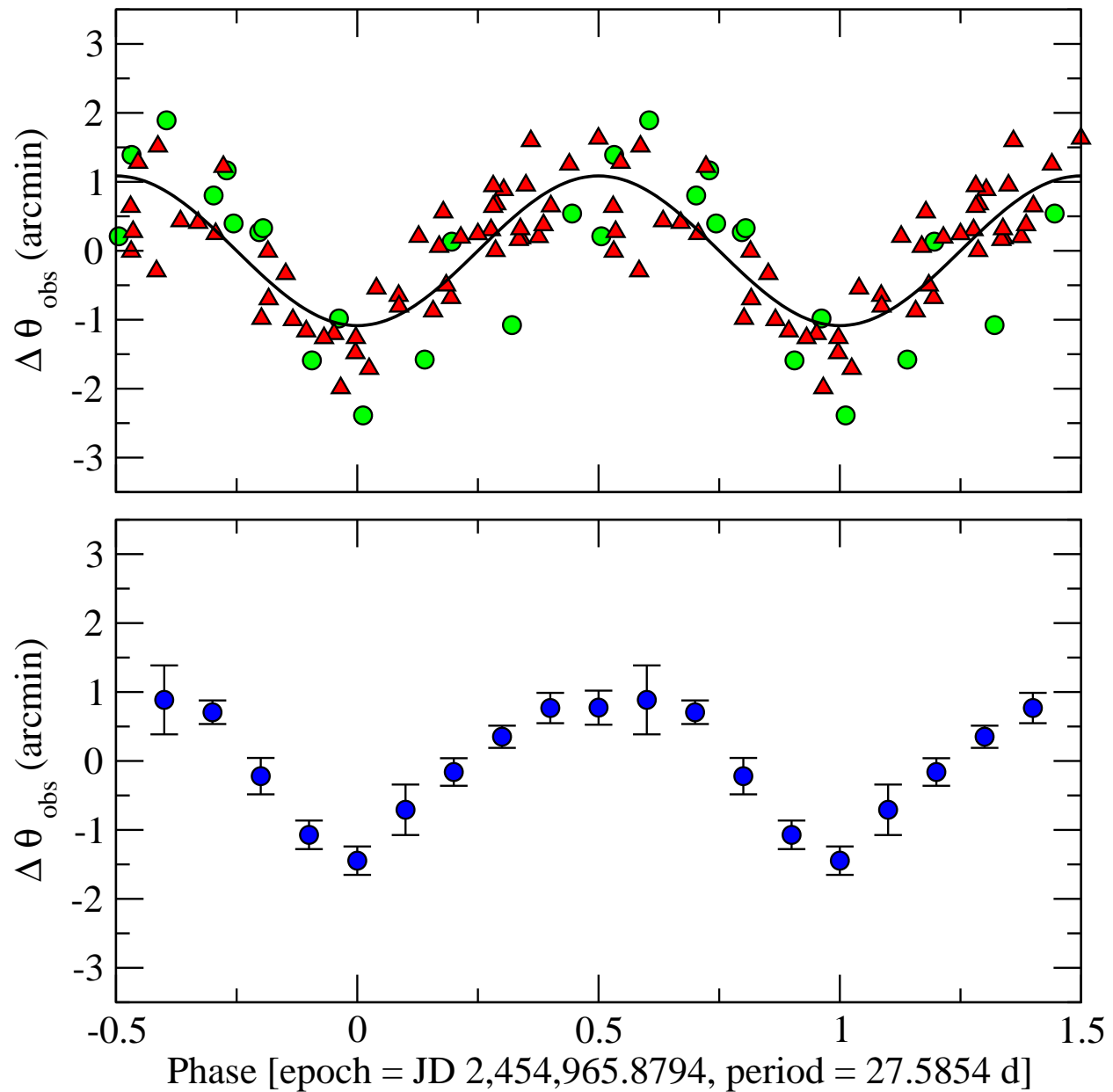


Fig. 4. After fitting a fourth order polynomial to 64 uncorrected measures of the Moon's angular diameter, we have determined the periodicity of the residuals. *Top*: individual measures. Triangles represent data taken during twilight and daytime. Dots represent data taken at night. *Bottom*: data binned by 0.1 in phase. The averages are based on 4 to 12 data points per bin.